HEAT CONDUCTION EQUATION AND RTD SELF HEATING

HEAT FLOW: Time response and other heat flow phenomena are governed by the Heat Conduction Equation. Solutions to the Heat Conduction Equation consist of a time independent final temperature distribution and a series sum of exponentially damped orthogonal functions which describe the evolution of the temperature distribution from the initial condition $f(x)$ to the final condition[1]. (Do not confuse the alpha used in this equation with the alpha used to describe an RTD's R-T curve.)

Heat Conduction Equation: $\alpha^2 \nabla^2 u = \frac{\partial u}{\partial t}$

$\alpha^2 = \frac{\kappa}{\rho s}$ Thermal Diffusivity (m²/s)

$\kappa =$ Thermal Conductivity (J/s • m • °C)

$\rho =$ Density (kg/m³)

$s =$ Specific Heat (J/°C)

Apply the Heat Conduction Equation to a thin film RTD mounted to a very thermally conductive, i.e. metal, surface, Figure 1 below. Since the RTD is very thin, approximate the problem as one-dimensional in x with a general solution $u(x, t)$ as shown below:

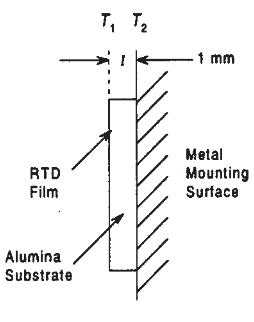

Figure 1

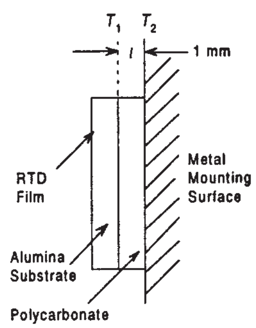

Figure 2

$$u(x,t) = (T_2 - T_1)\frac{x}{l} + T_1 + \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2\alpha^2 t/e^2} \sin\left[\frac{n\pi x}{l}\right]$$

$$b_n = \frac{2}{l}\int_0^l \left[f(x) - (T_2 - T_1)\frac{x}{l} - T_1\right] \sin\left[\frac{n\pi x}{l}\right]$$

$f(x) =$ Temperature distribution at time t = 0.

SELF HEATING: Once heat is introduced into the RTD by resistive heating, the equation which defines thermal conductivity must be satisfied:

$$j_u = -\kappa \frac{\partial u(T)}{\partial x}$$

Applying the conductivity equation as a boundary condition on the general solution for the RTD-on-a-surface example, $u(x, t)$ results in the self heating relationship:

$$\frac{P}{A} = -\alpha^2 u'(0)$$

P = Thermal power dissipated in the RTD = $V_+^2/R(T)$

A = Surface area of the RTD

Yielding our result:

$$\frac{P}{A} = -\alpha^2 \frac{(T_2 - T_1)}{l} \text{ or } T_1 = \frac{lP}{\alpha^2 A} + T_2 = \frac{lV_+^2}{\alpha^2 A\,R(T)} + T_2$$

Example 1: Applying the result to a low thermal impedance situation, examine an HEL-700 at 0°C, with 0.254 mm (0.010 in) thick alumina substrate (diffusivity k = 38 W/m°C) and 1000 Ω ice point resistance. Here the self heating error calculated from a 2.3 mA current is negligible, less than 0.02°C.

Example 2: Examining a high thermal impedance situation, use the same RTD, encapsulated in a plastic or epoxy package such as a TO-92. Approximating this as an intervening 1 mm thick layer of polycarbonate with diffusivity of 0.199 W/m°C, the 2.3 mA current now generates a 12.4°C offset.

A plastic encapsulated RTD will exhibit significantly greater temperature offset error than the same un-encapsulated RTD when both are mounted to a surface (or environment) with good thermal conductivity. However, for air measurement, the opposite occurs as the table illustrates!

TEMPERATURE OFFSET IN STILL AIR

RTD Current	Ceramic SIP	Encapsulated
0.1 mA	<0.02°C	<0.02°C
1.0 mA	0.83°C	0.50°C

Conclusion: When the thermal conductivity of the sensor packaging is lower than the thermal conductivity of the environment being measured, then the sensor packaging can increase self heating. More importantly, lower operating currents always reduce or eliminate self heating errors.

[1] Note that the constant α used in the Heat Conduction equation is different from the alpha used to describe a platinum RTD.